Role of Mathematics in Economics

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Introduction

Traditionally, application of mathematics had been restricted to the physical sciences, and the theories in the social sciences had been neglected, but in these days we notice that mathematical economics is flying high. We also observe that the articles on mathematical economics and fewer points on economic theory, occupy more prominent place in the economics journals.

Arguments given in the favor of mathematics look attractive in the first instance but they are not free of problems. It is argued by many economists that mathematical models are recognized in providing a rational approach to solving many of the problems in decision making, allocation, and forecasting[1]. Mathematical models present theoretical work in their own language, which is a tool of communication, but we know that a language must be simple and easily understood to be appreciated, but as a language, mathematics is not simple and easy. In this paper, an attempt is made to discourage the excessive use of mathematics in economics, by describing its drawbacks in the economic theory. Attention is given to the scientific nature of the economics.

In the beginning some definitions are given and comparison between different models are given, to develop an understanding in the subject.

Mathematisation of Economics

The major development of the second quarter of 20th century in the field of economics was the mathematization of economics. An economist of 19th century can not even understand the economic journals of present times.

Starting from the microeconomics theory, macroeconomics, international trade, economic development, public finance and all the other branches of economics have been changed into a number of equation. What and How did they do it?, is an important question to answer.

The development of mathematical economics is not revolutionary step. It took several centuries to develop the present stage of mathematical economics. Sir William Petty (1623-1687) is believed to be the first participant in this field. He used the terms of symbols in his studies, but he was not successful[2]. The first successful attempt was made by an Italian, named Giovanni Ceva (1647—1734) [3]. After these earlier developments, Antoine Augustin Cournot (1801-1877) made use of symbols in his theory of wealth.[4]

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[1] Lazarsfeld, Paul F. and Henry, Neil W., **Readings in Mathematical Social Science,** Chicago, Science Research Associates, Inc, (1966) PP. 1-2.
[2] Bell, John Fred, **A History of Economic Thought,** N.Y. Ronald Press Company, (1967), PP. 681-685.
[3] Same as above.
[4] Same as above.

After his work, Alfred Marshall in his *"Principles of Economics"* (1890), and Irving Fisher in his Ph.D. thesis *"Mathematical Investigations in The Theory of Value and Prices"* showed a great interest in mathematical formulation of the economic theory[5]. After their work, such a race began in this field that everyone specialised in mathematics and with less knowledge in the economic theory jumped in this new field and more and more articles started publishing with excessive use of mathematics and lacking theory.

Difference between Mathematical and Literary Economics

It is almost as hard to define mathematics as it is to define economics. An easy defination of economics is given by Jacob Viner, "Economics is what economists do"[6], so we can say that mathematics is what mathematicians do. Mathematical economics is not an individual branch of economics in the sense that international trade, public finance, or urban economics, but it is an approach to economic theory.

In mathematical economics, mathematical symbols and equation are used in the statement of the problem. Since mathematical economics is just an approach to economic analysis, it must not differ from the non-mathematical approach in the conclusion but we observe entirely opposite situation and here the problems starts. The major difference between mathematical economics and literary economics is that in the former, the assumptions and conclusions are described in mathematical symbols and equations whereas, in the later, words and sentences are used to achieve the desired goal[7].

Significance of Economics Models

The term "Model" is very common in economics. It can be defined as a set of assumptions from which the conclusions can be drawn. In simple words we can say that model is simply a representation of some aspects of the real world. Economic theory is descriptive as well as analytical. It does not give us complete descriptions of economic phenomenon, but by making certain assumptions, we can construct models. The models then help in representing reality and help in understanding the characteristics of economic behaviour.
In economic models, we can use both mathematical and theoretical approaches. The choice between these approaches depends upon the personal preference of the research person. If the model is math mathematical, it will consist of a set of symbols and equations, designed to described the structure of the model.
The mathematical models are severely criticized. The major criticism is that we know that economic model is merely a theoretical frame work and there is no major reason why it must be mathematical.

Limitations of Mathematical Models

Any body of mathematical knowledge must contain a main core of fundamental theories and these theories must be based and dependent on simple and general hypothetical models of the world of reality. A mathematical theory is not the result of generalization from direct observation because there is no such thing as direct observation in the mathematics.

Mathematical knowledge or mathematical language can just describe the economic phenomenon but can not explain it. In easy words, it describes what has been observed not what will be observed, so it lacks predictive power, which is the core of the "Positive Economics". We know that "The ultimate goal of a positive economics is the development of a theory or hypotheses that yields valid and meaningful predictions about the phenomenon not yet observed"[8].

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[5] Same as No. 2
[6] Galtung, Johan, **Theory and Methods of Social Research,** N.Y. Columbia University press, (1967), PP 97-98.
[7] Chiang, Alpha C. **Fundamental Methods of Mathematical Economics** 3rd ed., N.Y, McGraw Hill Co, (1984), PP. 3-4.
[8] Friedman, Milton, **Essays in Positive Economics,** Chicago, University of Chicago Press, (1953). PP,7.

Mathematical models are characteristically abstract and general, but it does not mean that they are universally valid. The simple point to state is that the more general and abstract a theory, the narrower its field of application. In summary, mathematically derived theories are dependent on the fundamental theories, they lack predictive power and they are not universal valid, so they are not superior to pure theoretical constructs.

A Critical Evaluation of Mathematical Formulation of Economic Theory

The emergence and spread of mathematical formulation of the economic theory started at the end of second world war, when economists felt quite confident in utilizing a separate knowledge of mathematics to relate to economic theory. This so-called mathematical revolution was noticed because of (1) shift of emphasis from partial to a general equilibrium analysis (2) interest in theories of growth, technical progress and other economic dynamics, (3) a revolution in the techniques of applied economics.[9]

This revolution divided the economists into four groups, i.e., mathematical economists, non-mathematical economists, applied economists and political economists. This division then divided the economic theory into different parts and served to misdirect the development in the theoretical field.

A major characteristic of the economic theory is its empirical nature. Empirical economics is about actual behavior, and the only reliable expression of this is observable, real world data. This empirical nature of the economics is used in testing the hypotheses but mathematical economics is non-empirical and so the non-empirical considerations can play a critical role in the testing of mathematical economics hypotheses. Some economists even the model builders themselves cannot sometime fully understand the mechanics of the large-scale mathematical models due to their complexity. This is another major draw back of the mathematical economics and a source of large criticism.

Another problem noticed in mathematical models is, they can only be used in large and heterogeneous collection of the variables but can not or just rarely be used in single variable, whereas the theoretical models are as good in single issue model as in the multiple variable studies.

In the case of developing large macroeconomic models for the economy, as it is done by the variety of individuals, and against a background of changing fashions in theory and mathematical practice, the conceptual basis for the model may not be consistent from one sector to another or from one generation of the equations to the other.

Another criticism is that mathematical modeling is a comparison with the interpreted results of the model with observations obtained from direct interaction with the real world, but the measurement of how closely the model fits in the real world is, in general, a problem which involves the full use of statistical techniques and in the absence of statistical techniques (which are a part of economic theory in one respect) mathematical model cannot precisely describe the behavior of a real-world phenomena. Some aspects are highlighted and others are neglected or perhaps ignored. So we can say that mathematical formulation of the theory is again dependent of the techniques of the theoretical economics. One of the strongest criticism against mathematical models is that sometimes a modeler finds that mathematical formulation of his problem is same as someone else's formulation.
To understand this problem, we take an example:-[10]

[9] Zarembka, Paul, **Frontiers in Econometrics,** N.Y, Academic Press, (1974), PP 1-2.
[10] Olinick, Micheael, **An Introduction to Mathematical Models in the Social and Life Sciences,** Massachusetts, Addision Wesley Publishing Company, (1978). PP.54-55.

Suppose a population made up of identical organisms, which reproduce at the same rate for every individual and are not affected by time. This model can be used to study many types of population, for example, increase in the number of scientists over time.

The mathematical model for this process is a first-order differential equation and can be written as:-

$$dP/dt = bP \text{ ----------} \quad (1)$$

Where P = P (t) is population and time (t). The (b) is the birthrate for each individual and is constant. Now suppose an opposite condition of pure death process, instead of pure birth process as in the last example. In this example it is assumed that no births occur and each individual has some positive death probability (d) at every moment. d is constant and called deathrate.

Mathematical equation has the form of:-

$$dP/dt = -dP \text{ ----------} \quad (2)$$

Now we take another example to describe a population in which both births and deaths occur. Birth rate is considered as (b) and death rate is (d) Both the death and birth rates are assumed to be positive constants and are independent of time, size of the population and the age of individual.

The model is differential equation and can be written as:-

$$dP/dt = (b-d)\,P \text{ ----------} \quad (3)$$

Setting $a = (b-d)$ we get

$$dP/dt = aP \text{ ----------} \quad (4)$$

We see in these examples that the mathematical formulation of three different theoretical models is the same, so we can say that use of mathematics in the economics can bring a lot of confusion by identifying the totally different problems in the same symbols and equation.

We notice that from couple of years the fraud in economics research is very common. I think that the reason for this problem is the same, as described above, because all the mathematical symbols and equations are the same and there is no identity of anybody's work, so it is too easy to convert any model according to our own needs, by changing the values of certain variables. We can well imagine the condition of mathematical economics that how it can construct a model for the whole economy which is a mix of thousands of variables, if it can not differenciate even a little example described above. The economic system is the outcome of centuries of human behaviour but mathematics have changed the easily understandable system into a complex, interdependent, multivariate equations process, which is unobservable in the real world.

Another problem is, that instead of constructing good theoretical models which could preserve the identity of hundred of variables working in an economy, all the journals are filled with mathematical equations which starts from irrelevant assumptions and state irrelevant theoretical conclusions[11].

We know that mathematical models begin with assumption but assumptions to be accepted, must be good enough for the approximation for the purpose in hand and if the theory works with these assumptions, then assumptions are said to be the good but in the mathematical models, assumptions can not be checked and are irrelevant. In this regard, I remember a funny story, there was a doctor, a chemist, and a mathematicians in a jungle. They had a can of food but nothing to open it. Doctor and chemist was thinking the possibility and alternative way to open the can, the mathematicians solved

[11] Leontief, W, "Academic Economics". Letters, **Science,** Feb, 25, 1983, .PP.904.

the problem by saying, assume we have a can opener. This story is a good example to understand the role of assumptions in mathematical models.

Among the well-known economists J.B. Say was the first economist who refused to accept mathematical formulated theories. He says, that economy is composed of a few fundamental principles and great number of conclusions drawn from these principles deduced from observations. It would be idle to imagine that greater precision or a more steady direction could be given to this study by the applications of mathematics to the solution of the problem.

Some Psychological Effects And Their Criticism

The following discussion contains some psychological and sociological effects of the mathematical revolution. By which some economists are influenced in favour or against it. There are some mathematical economists who insist on mathematical techniques just because of the fact that they are impressed by their professional appreciation by other mathematical economists. In recent years, we notice that number of universities have permitted their graduate students to substitute a knowledge of mathematics for a knowledge of one foreign language. I believe, the only reason for this change is not the importance of mathematics but the influence of some mathematical economists in the universities and the competition which has insisted the universities to keep pace in this race of mathematics and this has unhealthy effects on the economic theory.

Another point given sometime, in the favour of mathematical economics is that mathematics forces us into simplicity and mathematical formulations of economic theory are at times helpful to clarity and economy of expression[12].

In my opinion mathematical formulation of economic theory are at times unhelpful and it tends to hide simple and clear ideas behind complicated and hard techniques, and the thinking that only a large, simultaneous-equation model can describe the economy, is absolutely wrong and misinterpreted. And if it continue to happen we are in danger of becoming habitual of mathematical models, believing that actual world is like them.

The adoption of an irrational attitude by the economists in regarding mathematical models as superior to theoretical methods, has led to a situation where clear and intelligent ideas are artificially presented in the mathematical form, Just to show that author is intelligent, this thinking that economists with mathematical background can only make sense of real world is not right. I remember an advice of Samuelson that if anybody is a good theorist in economics but lacks knowledge in mathematics, he must not be depressed about his future in economic profession because some of the most distinguished economic theorists had no knowledge in economics.

Irving Fisher writes in his doctoral thesis that “As there is no place you can go by railroad that you cannot go a foot. So there is no place in the research we can go by mathematics that we cannot go without it”[13]. We can conclude that mathematics is neither a necessary nor a sufficient condition for a fruitful career in the economics.

It is believed that economic theories when presented in the mathematical form, are universally valid. I have already argued this is untrue not for only economics but for all sciences too, that total knowledge of anything, it may exist in some from, is outside the realm of science altogether.

I already stated that “Positive Economics” require economic theories to be testable and subject to tests, so we would have to accept the criterion of testability, but growing number of mathematically derived theories are simply untestable, then the question is, what should we believe in mathematical economics?

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[12] Katouzian, H., “Ideology and Methods in Economics”’ **Economica,** August, 1978, PP.370-372.
[13] Samuelson, P.A, **The Collected Scientific Papers,** Vol-II Massachusetts, M.I.T. Press. (1966), PP.691-692.

Before I close the topic, I like to make some more comments about the significance of the empirical reality as it is, or it may be conceived. We must insist on the essential unity of the two elements, "namely purely priori and purely empirical, for an intelligent perception of objective phenomena, and scientific strivings for the solution of problems with which they present mankind"[14]. This attitude can be clearly understood by the distinction of " Priori knowledge" and "Priori theory". 'Priori theory' is a logically or mathematical consistent model and it does not explain any knowledge of real phenomena by itself, but the "Priori knowledge", further depends on the assumptions which the theory makes and the events its predicts. In short it depends upon whether or not it is open to rational and empirical criticism.

I must conclude an important point at the end, as I have already mentioned, that abstraction is necessary for all scientific pursuits, This does not mean that every individual scientist does resort or want to resort to abstract analysis, but there are in limits to abstraction that is because mathematical theories are abstract and general, they cannot be universally valid[15], so, we see that mathematical economics neither begin with obsrvations nor a theory ends with conclusion and predictions so there is no need to select such approach.

[14] Katouzian, H, "Ideology and Methods in Economics"' **Economica,** August, 1978. PP.373.
[15] Same as above.

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